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Dual Ginzburg-Landau Theory for Quark Confinement and Dynamical Chiral-Symmetry Breaking⋆

H. Toki[a,b,c], H. Suganuma[a], S. Sasaki[b] and H. Ichie[c]

a) *The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-01, Japan*

b) *Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567, Japan*

c) *Department of Physics, Tokyo Metropolitan University, Hachiohji, Tokyo 192, Japan*

ABSTRACT

Nonperturbative features of QCD are studied using the dual Ginzburg-Landau (DGL) theory with QCD-monopoles. The linear quark potential appears in the QCD-monopole condensed vacuum. We find that QCD-monopole condensation plays an essential role to the dynamical chiral-symmetry breaking. We also investigate the QCD phase transition at finite temperature in the DGL theory.

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1. Introduction

The asymptotic freedom of QCD enables us to use the perturbative QCD calculation in the ultraviolet region as the deep inelastic scattering, but it leads to a strong coupling system in the infrared region of the hadron physics. This strong interaction provides color confinement and also dynamical chiral-symmetry breaking (DχSB) as the nonperturbative features of QCD. In particular, color confinement is one of the most unique features of the nonperturbative QCD, and therefore the understanding of the confinement mechanism is the central issue for the hadron physics. The DχSB is also an important feature in the nonperturbative QCD. Although recent lattice QCD studies support a close relation between color confinement and DχSB, no clear physical interpretation has been presented yet. In this paper, we investigate these nonperturbative properties of QCD in terms of the dual Ginzburg-Landau theory [1,2], an infrared effective gauge theory of QCD based on the dual Higgs mechanism.

2. Dual Ginzburg-Landau Theory

In 1981, 't Hooft presented an interesting fact that a non-abelian gauge theory reduces to an abelian gauge theory with color-magnetic monopoles by the abelian gauge fixing [3]. The appearance of these color-magnetic monopoles in QCD, called as QCD-monopoles, would be important for the confinement mechanism, because the quark confinement can be interpreted by condensation of such monopoles as shown below. Since QCD-monopoles behave as singularities of vector potential A_μ, we cannot introduce A_μ in the standard way. Instead, we can formulate the gauge theory without singularity following Zwanziger [4] by introducing the dual gauge field B_μ, which satisfies $\partial_\mu B_\nu - \partial_\nu B_\mu = {}^*F_{\mu\nu}$. In this formalism, the duality of the gauge theory becomes manifest. The QCD-monopole current k_μ couples with the dual gauge field as $k_\mu B^\mu$ in the similar way as the ordinary color-electric current, $j_\mu A^\mu$. The self-interaction among QCD-monopoles is introduced to realize QCD-monopole condensation, which is strongly supported by the recent studies based

on the lattice QCD [5].

The Lagrangian of the dual Ginzburg-Landau theory (DGL) is given as [1,2]

$$
\begin{aligned}
\mathcal{L}_{\rm DGL} = & -\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{A})]^\nu[n\cdot^*(\partial\wedge\vec{B})]_\nu+\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{B})]^\nu[n\cdot^*(\partial\wedge\vec{A})]_\nu \\
& -\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{A})]^2-\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{B})]^2+\bar{q}(i\partial\!\!\!/-e\vec{A}\!\!\!/\cdot\vec{H}-m)q \\
& +\sum_{\alpha=1}^{3}[|(i\partial_\mu-g\vec{\epsilon}_\alpha\cdot\vec{B}_\mu)\chi_\alpha|^2-\lambda(|\chi_\alpha|^2-v^2)^2]
\end{aligned}
\tag{1}
$$

with $\vec{A}^\mu=(A_3^\mu,A_8^\mu)$, $\vec{B}^\mu=(B_3^\mu,B_8^\mu)$ and $\vec{H}=(T_3,T_8)$. Here, g is the unit magnetic charge obeying the Dirac condition $eg=4\pi$, and $\vec{\epsilon}_\alpha$ denotes the relative magnetic charge of the QCD-monopole field χ_α (α=1,2,3). The dual Meissner effect is caused by QCD-monopole condensation due to the self-interaction of χ_α in the case $v^2>0$. It provides the mass of the dual gauge field B_μ. In the QCD-monopole condensed vacuum, the DGL Lagrangian becomes

$$
\begin{aligned}
\mathcal{L}_{\rm MF} = & -\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{A})]^\nu[n\cdot^*(\partial\wedge\vec{B})]_\nu+\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{B})]^\nu[n\cdot^*(\partial\wedge\vec{A})]_\nu \\
& -\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{A})]^2-\frac{1}{2n^2}[n\cdot(\partial\wedge\vec{B})]^2+\bar{q}(i\partial\!\!\!/-e\vec{A}\!\!\!/\cdot\vec{H}-m)q+\frac{1}{2}m_B^2\vec{B}^2
\end{aligned}
\tag{2}
$$

with $m_B=\sqrt{3}gv$. The QCD-monopole also becomes massive as $m_\chi=2\sqrt{\lambda}v$. Similar to the Higgs mechanism in the superconductivity, the color-electric field is then excluded in the QCD vacuum through the dual Meissner effect, and is squeezed between color sources to form the hadron flux tube.

3. Quark Confinement Potential

We investigate the quark confinement in terms of the linear inter-quark potential, which is supported by the lattice QCD in the quenched approximation. By integrating over A_μ and B_μ in the partition functional of the DGL theory, the

current-current correlation is obtained as

$$\mathcal{L}_j = -\frac{1}{2}\vec{j}_\mu D^{\mu\nu}\vec{j}_\nu \tag{3}$$

with the nonperturbative gluon propagator,

$$D_{\mu\nu} = \frac{1}{\partial^2}\{g_{\mu\nu} + (\alpha_e - 1)\frac{\partial_\mu\partial_\nu}{\partial^2}\} - \frac{1}{\partial^2}\frac{m_B^2}{\partial^2 + m_B^2}\frac{1}{(n\cdot\partial)^2}\epsilon^\lambda{}_{\mu\alpha\beta}\epsilon_{\lambda\nu\gamma\delta}n^\alpha n^\gamma\partial^\beta\partial^\delta \tag{4}$$

in the Lorentz gauge. Putting a static quark with color charge $\vec{Q}$ at $\mathbf{x} = \mathbf{a}$ and a static antiquark with color charge $-\vec{Q}$ at $\mathbf{x} = \mathbf{b}$, the quark current is written as $\vec{j}_\mu(x) = \vec{Q}g_{\mu0}\{\delta^3(\mathbf{x} - \mathbf{b}) - \delta^3(\mathbf{x} - \mathbf{a})\}$. We finally obtain the inter-quark potential including the Yukawa and linear terms,

$$V(r) = -\frac{\vec{Q}^2}{4\pi}\frac{e^{-m_B r}}{r} + \frac{\vec{Q}^2 m_B^2}{8\pi}\ln(\frac{m_B^2 + m_\chi^2}{m_B^2})\cdot r, \tag{5}$$

where $r = |\mathbf{r}| = |\mathbf{b} - \mathbf{a}|$ is the relative distance. Here, we have identified $\mathbf{n}//\mathbf{r}$, which is also used in the similar context of the dual string theory [6], because of the axial symmetry of the system and the energy minimum condition. Otherwise, the energy of the system diverges. It should be noted that the expression for the string tension, the coefficient of the linear potential, agrees with the one for the energy per length of the vortex in the type-II superconductor.

We compare the static potential with the phenomenological one, for example, the Cornell potential. We get a good agreement as shown in Fig.1 with the choice of $e = 5.5$, $m_B = 0.5$GeV and $m_\chi = 1.26$GeV corresponding to $\lambda = 25$ and $v = 126$MeV, which provide k=1.0GeV/fm for the string tension and the radius of the hadron flux as $m_B^{-1} = 0.4$fm.

4. Dynamical Chiral-Symmetry Breaking

The dynamical chiral-symmetry breaking (DχSB) is also important for the hadron properties as well as color confinement. We discuss here the DχSB in terms of the mass generation of light quarks in the QCD-monopole condensed vacuum [1,7]. We formulate the Schwinger-Dyson (SD) equation for massless quark as

$$S_q^{-1}(p) = \not{p} + \int \frac{d^4k}{i(2\pi)^4} \vec{Q}^2 \gamma^\mu S_q(k) \gamma^\nu D_{\mu\nu}^{\rm sc}(k-p), \tag{6}$$

where we assume the quark propagator $S_q(p)$ as $S_q(p)^{-1} = \not{p} - M(-p^2) + i\eta$. In the presence of light quarks, there appears the screening effect in the long distance of the linear potential corresponding to the cut of the hadron flux tube by the light-quark pair creation. Hence, we introduce the corresponding infrared cutoff a in the gluon propagator $D_{\mu\nu}^{\rm sc}$ by the replacement, $\frac{1}{(n\cdot k)^2} \rightarrow \frac{1}{(n\cdot k)^2 + a^2}$ [1].

Taking the trace and making the Wick rotation in the k_0-plane, we obtain the SD equation in the Euclidean metric,

$$M(p^2) = \int \frac{d^4k}{(2\pi)^4} \vec{Q}^2 \frac{M(k^2)}{k^2 + M^2(k^2)} D_\mu^{\mu {\rm sc}}(k-p), \tag{7}$$

where the gluon propagator including the screening effect is given as

$$D_\mu^{\mu {\rm sc}}(k) = \frac{1}{(n\cdot k)^2 + a^2} \cdot \frac{1}{k^2} \cdot \frac{2m_B^2}{k^2 + m_B^2} \{k^2 - (n\cdot k)^2\} + \frac{3+\alpha_e}{k^2} \tag{8}$$

in the Lorentz gauge. After performing the angular integral, we obtain the final expression for the SD equation,

$$\begin{aligned} M(p^2) = \int_0^\infty \frac{dk^2}{16\pi^2} \frac{\vec{Q}^2 M(k^2)}{k^2 + M^2(k^2)} \Bigg(& \frac{4k^2}{k^2 + p^2 + m_B^2 + \sqrt{(k^2+p^2+m_B^2)^2 - 4k^2p^2}} \\ & + \frac{(1+\alpha_e)k^2}{\max(k^2, p^2)} + \frac{1}{\pi p_T} \int_{-k}^{k} dk_n \frac{1}{\tilde{k}_n^2 + a^2} \\ & \times [(m_B^2 - a^2) \ln\{\frac{\tilde{k}_n^2 + (k_T + p_T)^2 + m_B^2}{\tilde{k}_n^2 + (k_T - p_T)^2 + m_B^2}\} + a^2 \ln\{\frac{\tilde{k}_n^2 + (k_T+p_T)^2}{\tilde{k}_n^2 + (k_T - p_T)^2}\}] \Bigg) \end{aligned} \tag{9}$$

with $\tilde{k}_n \equiv k_n - p_n$ and $k_T \equiv (k^2 - k_n^2)^{1/2}$.

In solving the SD equation, we use the Higashijima-Miransky ansatz with a hybrid type of the running coupling constant,

$$\tilde{e} = e(\max\{p^2, k^2\}), \quad e^2(p^2) = \frac{48\pi^2(N_c+1)}{(11N_c - 2N_f)\ln\{(p^2+p_c^2)/\Lambda_{\rm QCD}^2\}}. \tag{10}$$

Here, p_c is defined as $p_c \equiv \Lambda_{\rm QCD} \exp[\frac{24\pi^2}{e^2} \cdot \frac{N_c+1}{11N_c-2N_f}]$ with $e = e(0)$. This ansatz naturally connects to the asymptotic freedom of the running coupling at large momentum. The coupling constant at low energy, $e(p^2 \sim 0) \simeq e$, controls the strength of the linear confinement potential.

Fig.2 shows the quark mass function $M(p^2)$ with e=5.5 and $a = 80$MeV. The QCD scale parameter is set to a realistic value $\Lambda_{\rm QCD} = 200$MeV. In order to see the effect of QCD-monopole condensation, we vary the mass of the dual gauge field, m_B. There is no non-trivial solution for the case with small $m_B < 300$MeV. A non-trivial solution is barely obtained at $m_B = 300$MeV, and $M(p^2)$ increases rapidly with m_B as shown in Fig.2. Hence, QCD-monopole condensation provides a crucial contribution to DχSB.

Taking the value for the mass of the dual gauge field as $m_B = 0.5$GeV extracted from the linear potential, we get the result for $M(p^2)$ as shown in Fig.3. The quark mass function $M(p^2)$ in the space-like region is directly obtained from the SD equation. We extrapolate $M(p^2)$ into the time-like region using a polynomial function as a simulation of the analytic continuation. This curve does not cross the on-shell condition $M^2(p^2) + p^2 = 0$ (p_μ: Euclidean momentum) and hence the quark propagator does not have a physical pole. This may indicate the light-quark confinement.

We calculate the several quantities related to DχSB from the solution of the SD equation. The constituent quark mass in the infrared region is found to be $M(0)$=348MeV. The quark condensate is obtained as $\langle \bar{q}q \rangle = -(229\text{MeV})^3$. The pion decay constant is also calculated as f_π=83.6MeV using the Pagels-Stoker formula [9]. These values are to be compared with the standard values; $M(0)$=350 MeV, $\langle \bar{q}q \rangle = -(225 \pm 50\text{MeV})^3$ and $f_\pi = 93$MeV.

5. QCD Phase Transition at Finite Temperature

The DGL theory is now able to describe many interesting quantities. Here, we study the change of the QCD vacuum at finite temperature [10] in terms of QCD-monopole condensation. To concentrate on the confinement properties, we consider the pure gauge case, where the quark degrees of freedom are frozen. In this case, we can drop the quark term in the DGL Lagrangian and perform integration over the gauge field A_μ. Hence, we obtain the partition functional as

$$Z[J] = \int \mathcal{D}\chi_\alpha \mathcal{D}\vec{B}_\mu \exp\left(i \int d^4x \{\mathcal{L}_{\rm DGL} - J\sum_{\alpha=1}^{3} |\chi_\alpha|^2\} \right), \tag{11}$$

where $\mathcal{L}_{\rm DGL}$ has a simple form,

$$\mathcal{L}_{\rm DGL} = -\frac{1}{4}(\partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu)^2 + \sum_{\alpha=1}^{3}[|(i\partial_\mu - g\vec{\epsilon_\alpha}\cdot\vec{B}_\mu)\chi_\alpha|^2 - \lambda(|\chi_\alpha|^2 - v^2)^2]. \tag{12}$$

Here, we have introduced the quadratic source term instead of the linear source term, which is commonly used. Such an introduction of the quadratic source term is quite powerful for the formulation of the effective potential, especially in the negative-curvature region of the classical potential, where the use of the linear source term does not work well.

The effective potential at finite temperature, which physically corresponds to the thermodynamical potential, is then obtained as

$$\begin{aligned} V_{\rm eff}(\bar\chi; T) = 3\lambda(\bar\chi^2 - v^2)^2 &+ 3\frac{T}{\pi^2}\int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2+m_B^2}/T}\right) \\ &+ \frac{3}{2}\frac{T}{\pi^2}\int_0^\infty dk k^2 \ln\left(1 - e^{-\sqrt{k^2+m_\chi^2}/T}\right). \end{aligned} \tag{13}$$

Here, the masses of the QCD-monopole and the dual gauge field depend on the

QCD-monopole condensate $\bar{\chi}$,

$$m_\chi^2(\bar{\chi}) = 2\lambda(3\bar{\chi}^2 - v^2) + J(\bar{\chi}) = 4\lambda\bar{\chi}^2, \quad m_B^2(\bar{\chi}) = 3g^2\bar{\chi}^2. \tag{14}$$

We provide the calculated results on the effective potential in Fig.4. At $T = 0$, one minimum appears at a finite $\bar{\chi}$, which corresponds to the condensed phase of QCD-monopoles. As the temperature increases, the minimum moves toward a small $\bar{\chi}$ value, and the second minimum appears at $\bar{\chi} = 0$ above the lower critical temperature $T_{\rm low} \simeq 0.39$GeV. The potential values at the two minima become equal at $T_c \simeq 0.49$GeV, which corresponds to the thermodynamical critical temperature for the QCD phase transition. In this case, it is of first order. Then the trivial vacuum stays as the absolute minimum above the critical temperature. When light dynamical quarks are included, we also expect the chiral-symmetry restoration as well as the deconfinement phase transition at this critical temperature, because QCD-monopole condensation is essential for DχSB as demonstrated in the previous sections.

Since the critical temperature seems too high in the above discussion, we consider the temperature dependence of the monopole coupling constant λ. This is very probable because the interaction among QCD-monopoles is weakened at finite temperature due to the asymptotic free behavior of QCD. Hence, we examine a simple case where λ decreases linearly with T,

$$\lambda(T) = \lambda\left(\frac{T_c - \alpha T}{T_c}\right), \tag{15}$$

where a constant $\alpha = 0.96$ is chosen so as to satisfy $T_c = 0.2$GeV. We find in this case also a weak first order phase transition. Here, we are able to compare with the numerical results with the pure-gauge lattice QCD on the string tension $k(T)$ [11]. The lattice QCD results are shown by black dots below the critical temperature in Fig.5, while the results for the variable $\lambda(T)$ go through the dots. We also find that the masses of glueballs (QCD-monopoles, the dual gauge fields)

drop largely toward T_c with T from those of order of 1 GeV at zero temperature. It would be very interesting to check this phenomena by the lattice QCD and also by experiment.

6. Summary

We have studied the dual Ginzburg-Landau (DGL) theory as the effective theory of QCD. QCD is reduced to an abelian gauge theory with QCD-monopoles in 't Hooft's abelian gauge. According to QCD-monopoles condensation, the dual Higgs mechanism works as the mass generation of the dual gauge field. We have derived the static inter-quark potential in the DGL theory. In the QCD-monopole condensed vacuum, there appears the linear potential responsible for the quark confinement.

We have also studied the dynamical chiral-symmetry breaking (DχSB) in the DGL theory. We find an essential role of QCD-monopole condensation to DχSB. With the parameters extracted from the quark confining potential, we obtain reasonable values for the constituent quark mass $M(p=0)$=348MeV, the quark condensate $\langle \bar{q}q \rangle = -(229\mathrm{MeV})^3$ and the pion decay constant f_π=83.6MeV.

The DGL theory predicts the existence of an axial-vector particle B_μ and the scalar QCD-monopole χ_α with masses of $m_B \sim 0.5$GeV and $m_\chi \sim 1.5$GeV with admittedly a large error of about 1GeV. It would be important to look for these particles in the hadron spectra. Theoretically, we are investigating the decay properties of these particles.

We have discussed also the properties of the QCD vacuum at finite temperature in terms of the DGL theory. We find a first order deconfinement phase transition in the pure gauge case. We are now making an effort to introduce dynamical quarks in the discussion of the QCD phase transition.

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Figure Captions

Fig.1. The static quark potential $V(r)$ in the dual Ginzburg-Landau theory. The dashed curve denotes the Cornell potential.

Fig.2. The dynamical quark mass $M(p^2)$ as a function of the Euclidean momentum squared p^2 for m_B=300, 400 and 500 MeV.

Fig.3. The dynamical quark mass squared $M^2(p^2)$ as a function of p^2. The dotted straight line denotes the on-shell state.

Fig.4. The effective potentials at various temperatures as functions of the QCD-monopole condensate $\bar{\chi}$. The crosses denote their minima.

Fig.5. The string tensions $k(T)$ as functions of the temperature T for a constant λ and a variable $\lambda(T)$. The lattice QCD results in the pure gauge are shown by black dots.